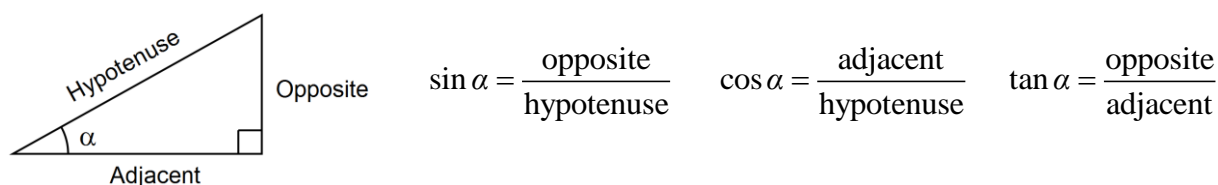


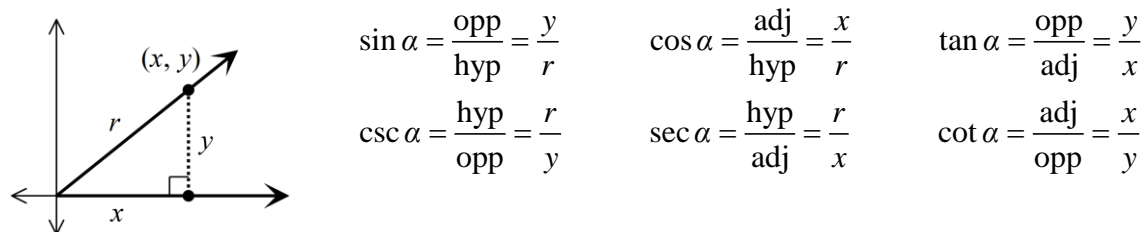
## 6.4 The Trigonometric Functions

The six trigonometric functions are the sine (sin), cosine (cos), tangent (tan), cosecant (csc), secant (sec), and cotangent (cot) functions. There are several ways to define these functions of trigonometry. One of the most common mnemonic devices is SOH-CAH-TOA.



All of these ratios can be written in terms of an angle in the coordinate plane.

If  $(x, y)$  is any point other than the origin on the terminal side of an angle  $\alpha$  in standard position and  $r = \sqrt{x^2 + y^2}$ , then



**Reciprocal Identities:**

$$\csc \alpha = \frac{1}{\sin \alpha} \quad \sec \alpha = \frac{1}{\cos \alpha} \quad \cot \alpha = \frac{1}{\tan \alpha}$$

The *signs* of the trigonometric functions depend on the quadrant in which the angle lies and the corresponding signs of  $x$  and  $y$  (remember  $r$  is always positive).

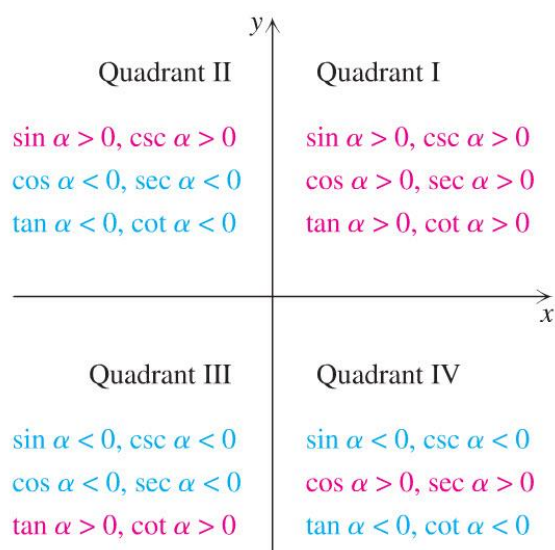
A good mnemonic to remember which functions are positive in each quadrant is “**All Students Take Calculus**”:

Quadrant I: All of them are positive

Quadrant II: sin and csc are positive

Quadrant III: tan and cot are positive

Quadrant IV: cos and sec are positive



### The Pythagorean Identity

In the previous section we used the fact that  $P(x, y) = (\cos(\theta), \sin(\theta))$  lies on the Unit Circle,  $x^2 + y^2 = 1$ . If we substitute  $x = \cos(\theta)$  and  $y = \sin(\theta)$  into  $x^2 + y^2 = 1$ , we get the identity  $(\cos(\theta))^2 + (\sin(\theta))^2 = 1$ . This is the Pythagorean Identity: For any angle  $\theta$ ,  $\cos^2(\theta) + \sin^2(\theta) = 1$ .

If we know the sine or cosine of an angle, then we can use the *Pythagorean Identity* to find the value of the other function of the angle. (Note: you can also figure this out by drawing a triangle and using the Pythagorean Theorem).

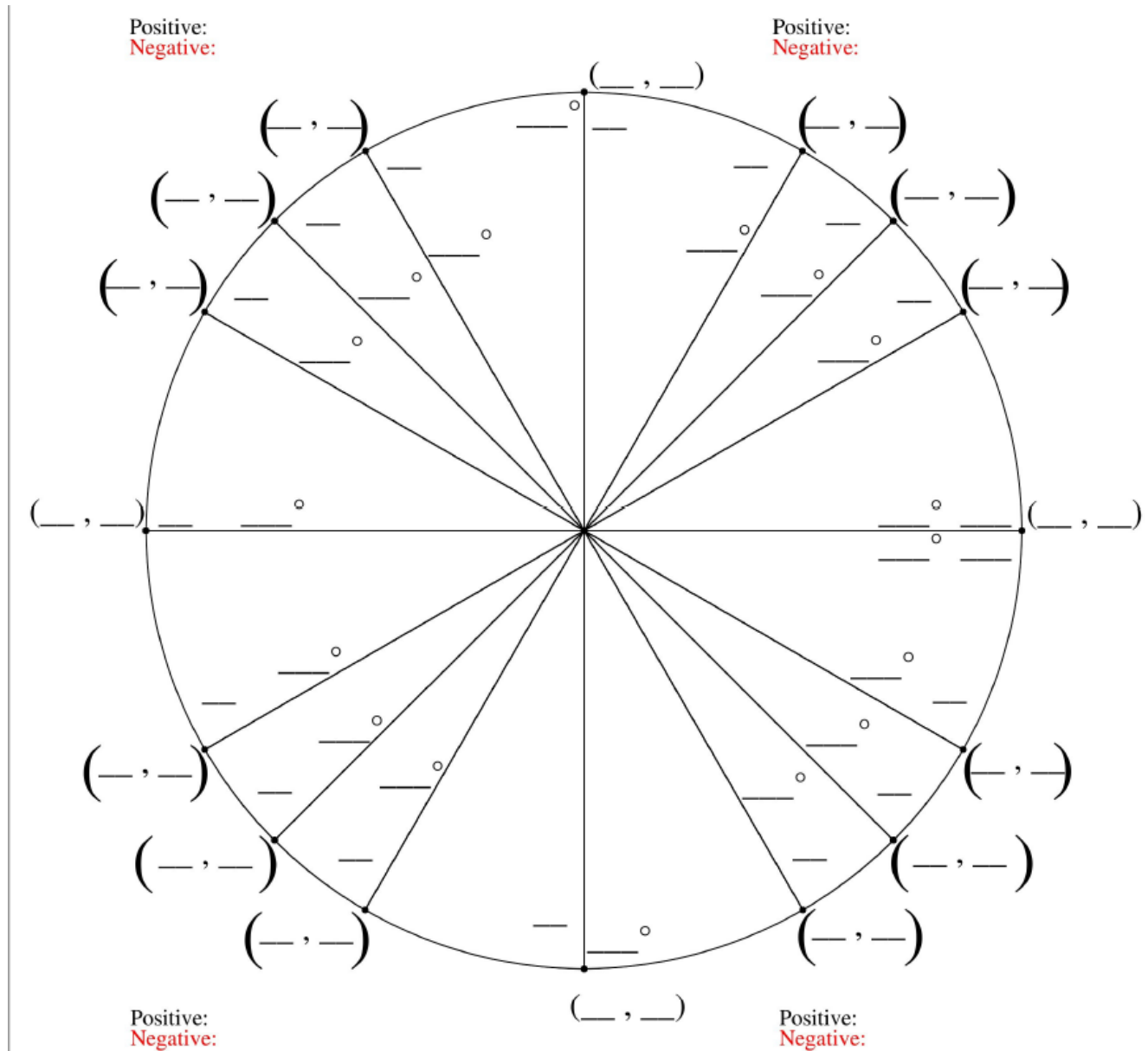
**Example:** Find  $\sin \theta$  given that  $\cos \theta = 1/4$  and  $\theta$  is in Quadrant I.

**Example:** Find  $\cos \theta$  given that  $\sin \theta = -\sqrt{5}/3$  and  $\theta$  is in Quadrant III.

### Examples:

Find the values of the six trigonometric functions of the angle  $\alpha$  in standard position whose terminal side passes through  $(-2, -4)$ .

Let's fill out the remaining portion of the Unit Circle keeping the special right triangles in mind!



**Examples:** Find the exact values of the following:

1.  $\sin 0^\circ$

2.  $\cos \pi$

3.  $\tan \left(-\pi/2\right)$

4.  $\csc \left(-270^\circ\right)$

5.  $\sin \left(\pi/4\right)$

6.  $\cos \left(-225^\circ\right)$

7.  $\cot \left(5\pi/4\right)$

8.  $\sec 315^\circ$

9.  $\sin 30^\circ$

10.  $\cos (7\pi/6)$

11.  $\tan (-\pi/3)$

12.  $\csc 150^\circ$

13.  $\cot (-240^\circ)$

14.  $\sec(-\pi/6)$

15.  $\cos (5\pi/3)$

16.  $\tan (-150^\circ)$